Spin ordered phase transitions in isospin asymmetric nuclear matter

A. A. Isayev

Kharkov Institute of Physics and Technology, Academicheskaya Street 1, Kharkov, 61108, Ukraine

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Spin polarized states in nuclear matter with Skyrme effective forces are studied on the base of a Fermi liquid theory for a wide range of isospin asymmetries and densities. There are a few possible scenarios of spin ordered phase transitions: (a) nuclear matter with SLy4 interaction undergoes at some critical density a phase transition to a spin polarized state with the oppositely directed spins of neutrons and protons; (b) for SkI5 interaction, a spin polarized state with the like-directed neutron and proton spins is formed; (c) nuclear matter with SkI3 interaction under increasing density, at first, undergoes a phase transition to the state with the opposite directions of neutron and proton spins, which goes over at larger density to the state with the same direction of nucleon spins. Spin polarized states at strong isospin asymmetry are accompanied by the long tails in the density profiles of the neutron spin polarization parameter near the critical density, if the energy gain of the transition from the nonpolarized state to a polarized one is the decreasing function of isospin asymmetry (SLy4 force). If the energy gain is increased with isospin asymmetry, there are no long tails in the density distribution of the neutron spin polarization parameter (SkI3, SkI5 forces).

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The issue of spontaneous appearance of spin polarized states in nuclear matter is a topic of a great current interest due to its relevance in astrophysics. In particular, the scenarios of supernova explosion and cooling of neutron stars are essentially different, depending on whether nuclear matter is spin polarized or not. On the one hand, the models with the effective Skyrme and Gogny nucleon-nucleon (NN) interaction predict the occurrence of spin instability in nuclear matter at densities in the range from ϱ_0 to $6\varrho_0$ for different parametrizations of the NN potential [1]–[14] ($\varrho_0 = 0.16\,\mathrm{fm}^{-3}$ is the nuclear saturation density). On the other hand, for the models with the realistic NN interaction, the ferromagnetic phase transition seems to be suppressed up to densities well above ϱ_0 [15]–[21].

Here we continue the research of spin polarizability of nuclear matter with the use of an effective NN interaction. The main objective is to study the possible scenarios of spin ordered phase transitions in nuclear matter with Skyrme forces, attracting parametrizations of a NN potential being relevant for calculations at strong isospin asymmetry and high density. In particular, we choose SLy4 effective interaction, constructed originally to reproduce the results of microscopic neutron matter calculations [22]. We utilize SkI3 and SkI5 parametrizations as well, giving a correct description of isotope shifts in neutron-rich medium and heavy nuclei [23]. As compared with the research of Ref. [12] with SLy4 and SLy5 effective interactions, here we explore a wider domain of isospin asymmetries, including symmetric nuclear matter and neutron matter as limiting cases. Besides, we use also SkI3 and SkI5 parametrizations, that will allow us to study new scenarios of spin ordered phase transitions, not found in Ref. [12].

The basic formalism is presented in detail in Ref. [12]. We are interested in studying spin polarized states with like-directed and oppositely directed spins of neutrons and protons. One should solve the self-consistent equations for the coefficients $\xi_{00},\xi_{30},\xi_{03},\xi_{33}$ in the expansion of the single particle energy in Pauli matrices in spin and isospin spaces

$$\begin{aligned}\xi_{00}(\mathbf{p}) &= \varepsilon_0(\mathbf{p}) + \tilde\varepsilon_{00}(\mathbf{p}) - \mu_{00},\ \xi_{30}(\mathbf{p}) = \tilde\varepsilon_{30}(\mathbf{p}),\\ \xi_{03}(\mathbf{p}) &= \tilde\varepsilon_{03}(\mathbf{p}) - \mu_{03},\ \xi_{33}(\mathbf{p}) = \tilde\varepsilon_{33}(\mathbf{p}).\end{aligned} \tag{1}$$

Here $\varepsilon_0(\mathbf{p})$ is the free single particle spectrum, and $\tilde\varepsilon_{00},\tilde\varepsilon_{30},\tilde\varepsilon_{03},\tilde\varepsilon_{33}$ are the Fermi liquid (FL) corrections to the free single particle spectrum, related to the normal FL amplitudes $U_0(\mathbf{k}),...,U_3(\mathbf{k})$ by formulas

$$\begin{aligned}\tilde\varepsilon_{00}(\mathbf{p}) &= \frac{1}{2\mathcal{V}}\sum_{\mathbf{q}} U_0(\mathbf{k}) f_{00}(\mathbf{q}),\ \mathbf{k} = \frac{\mathbf{p}-\mathbf{q}}{2},\\ \tilde\varepsilon_{30}(\mathbf{p}) &= \frac{1}{2\mathcal{V}}\sum_{\mathbf{q}} U_1(\mathbf{k}) f_{30}(\mathbf{q}),\\ \tilde\varepsilon_{03}(\mathbf{p}) &= \frac{1}{2\mathcal{V}}\sum_{\mathbf{q}} U_2(\mathbf{k}) f_{03}(\mathbf{q}),\\ \tilde\varepsilon_{33}(\mathbf{p}) &= \frac{1}{2\mathcal{V}}\sum_{\mathbf{q}} U_3(\mathbf{k}) f_{33}(\mathbf{q}).\end{aligned} \tag{2}$$

The distribution functions $f_{00}, f_{03}, f_{30}, f_{33}$, in turn, can be expressed in terms of the components ξ of the single particle energy and satisfy the normalization conditions for the total density $\varrho_n + \varrho_p = \varrho$, excess of neutrons over protons $\varrho_n - \varrho_p \equiv \alpha\varrho$, ferromagnetic (FM) $\varrho_\uparrow - \varrho_\downarrow \equiv \Delta\varrho_{\uparrow\uparrow}$ and antiferromagnetic (AFM) $(\varrho_{n\uparrow} + \varrho_{p\downarrow}) - (\varrho_{n\downarrow} + \varrho_{p\uparrow}) \equiv \Delta\varrho_{\uparrow\downarrow}$ spin order parameters, respectively (α being the isospin asymmetry parameter, $\varrho_\uparrow = \varrho_{n\uparrow} + \varrho_{p\uparrow}$ and $\varrho_\downarrow = \varrho_{n\downarrow} + \varrho_{p\downarrow}$, with $\varrho_{n\uparrow}, \varrho_{n\downarrow}$ and $\varrho_{p\uparrow}, \varrho_{p\downarrow}$ being the neutron and proton number densities with spin up and spin down). The quantities of interest are the neutron and proton spin polarization parameters

$$\Pi_n = \frac{\varrho_{n\uparrow} - \varrho_{n\downarrow}}{\varrho_n}, \quad \Pi_p = \frac{\varrho_{p\uparrow} - \varrho_{p\downarrow}}{\varrho_p},$$

characterizing spin ordering in neutron and proton subsystems. In the numerical solution of the self-consistent equations we utilize SLy4, SkI3 and SkI5 Skyrme forces. Note that the density dependence of the nuclear symmetry energy, calculated with these Skyrme interactions, gives the neutron star models in a broad agreement with the observables [24]. Another important constraint on the set of Skyrme force parameters can be obtained, if to consider expression for the effective mass of a neutron m_n^* in totally spin polarized neutron matter

$$\frac{m_0}{m_n^*} = 1 + \frac{\varrho m_0}{\hbar^2} t_2(1 + x_2), \tag{3}$$

where m_0 is the bare mass of a nucleon. Eq. (3) follows from expressions for FL amplitudes in neutron matter [12]. Since usually for Skyrme parametrizations $t_2 < 0$, then we get the constraint $x_2 \leq -1$, which guarantees the stability of totally polarized neutron matter at high densities [10, 14]. The Skyrme parametrizations SLy4, SkI3 and SkI5 satisfy this condition.

Fig. 1a shows the density dependence of the neutron and proton spin polarization parameters at zero temperature for SLy4 force. The main qualitative feature is that for SLy4 force there are only solutions corresponding to the oppositely directed spins of neutrons and protons in a spin polarized state and there are no solutions corresponding to the same direction of neutron and proton spins. The reason is that for SLy4 force the FL amplitude U_1, determining spin–spin correlations, becomes more and more repulsive with the increase of nuclear matter density, while the FL amplitude U_3, describing spin–isospin correlations, becomes more and more attractive with density, leading to spin instability with the oppositely directed spins of neutrons and protons at higher densities. The critical density of spin instability in symmetric nuclear matter ($\alpha = 0$), corresponding to AFM spin ordering ($\Delta\varrho_{\uparrow\downarrow} \neq 0$, $\Delta\varrho_{\uparrow\uparrow} = 0$), is $\varrho_c \approx 0.33$ fm^{-3}. It is less than the critical density of FM instability in neutron matter, $\varrho_c \approx 0.59$ fm^{-3}. Even small admixture of protons to neutron matter leads to the appearance of long tails in the density profiles of the neutron spin polarization parameter near the transition point to a spin ordered state. As a consequence, a spin polarized state is formed much earlier in density than in pure neutron matter.

As seen from Fig. 1b, for SkI5 force, oppositely to the case with SLy4 force, there are only solutions corresponding to the same direction of neutron and proton spins in a polarized state and there are no solutions corresponding to their opposite directions. Comparing to the previous case, the FL amplitudes U_1 and U_3 exchange their roles: the FL amplitude U_3 becomes more and more repulsive with the increase of nuclear matter density, while the FL

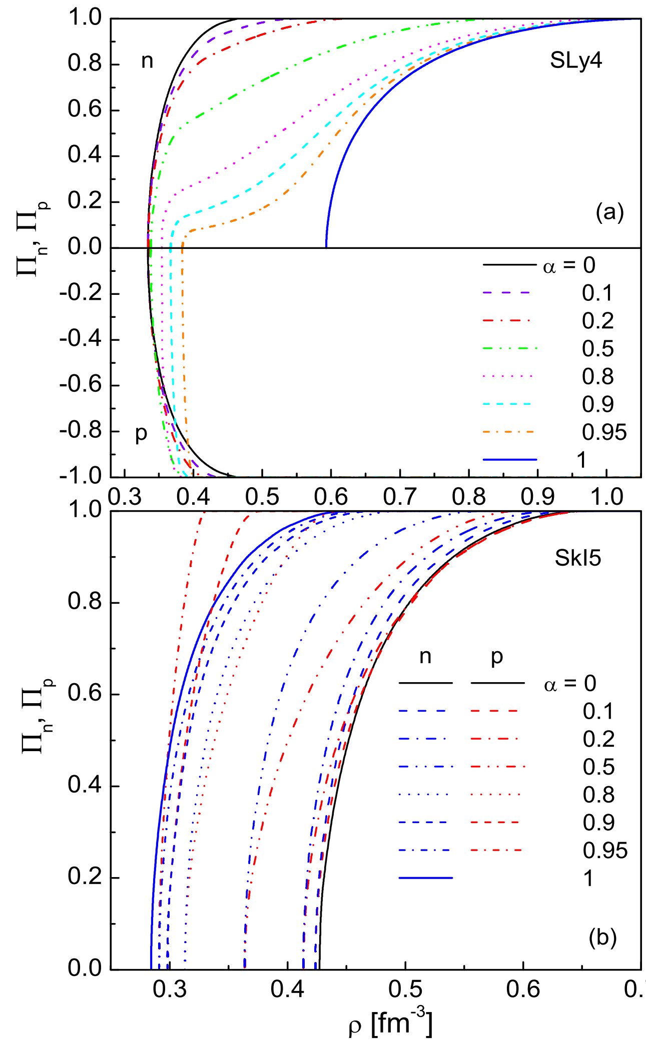

FIG. 1: (Color online) Neutron and proton spin polarization parameters as functions of density at zero temperature for (a) SLy4 force and (b) SkI5 force.

amplitude U_1 becomes more and more attractive with density, leading to spin instability with the like-directed spins of neutrons and protons at higher densities. For SkI5 force, a phase transition to the FM spin state in neutron matter takes place at the critical density $\varrho_c \approx 0.28$ fm^{-3}. It is less than the critical density of spin instability in symmetric nuclear matter $\varrho_c \approx 0.43$ fm^{-3}, corresponding to FM spin ordering ($\Delta\varrho_{\uparrow\uparrow} \neq 0$, $\Delta\varrho_{\uparrow\downarrow} = 0$). An important peculiarity is that there are no long tails in the density profiles of the neutron spin polarization parameter at large isospin asymmetry. Hence, in this case a small admixture of protons to neutron matter doesn't considerably change the critical density of spin instability and even leads to its increase.

In Fig. 2 the total energies per nucleon of the spin ordered and nonpolarized states are compared at zero temperature for SLy4 and SkI5 forces. One can see that

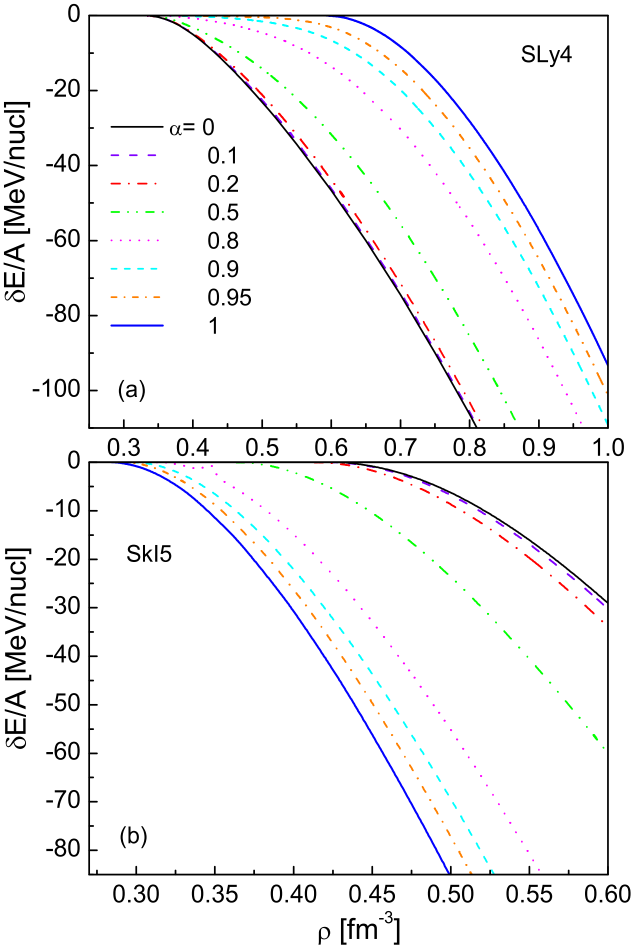

FIG. 2: (Color online) Total energy per nucleon, measured from its value in the normal state, as a function of density at zero temperature for (a) SLy4 force and (b) SkI5 force.

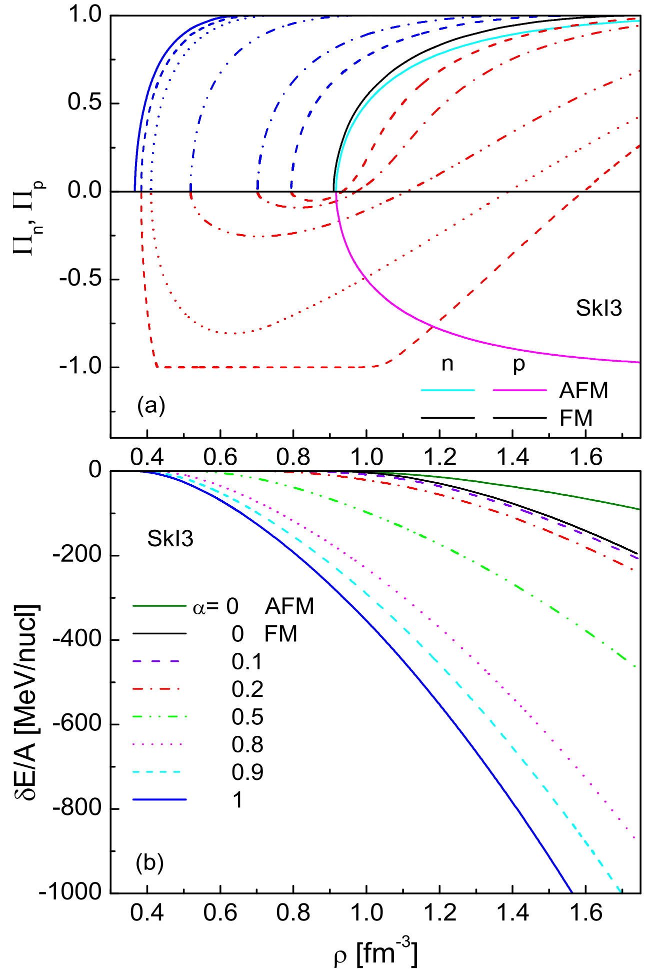

FIG. 3: (Color online) The dependences for SkI3 force: (a) Same as in Fig. 1 with the legends of Fig. 1b; (b) same as in Fig. 2. Also the curves, corresponding to FM and AFM spin ordering in symmetric nuclear matter, are shown.

nuclear matter undergoes a phase transition to the state with the oppositely directed (SLy4 force) or like-directed (SkI5 force) spins of neutrons and protons at some critical density, depending on isospin asymmetry. It is worth to note an important difference in isospin dependences for these two cases. For SLy4 interaction, the difference between the total energies of spin polarized and nonpolarized states is largest at the given density for symmetric nuclear matter while for SkI5 interaction it is largest for neutron matter. This means that a phase transition in density to a spin polarized state will take place earlier in symmetric nuclear matter than in neutron matter for SLy4 force, while for SkI5 force, oppositely, it will occur earlier in neutron matter, as was mentioned above.

Fig. 3a shows the neutron and proton spin polarization parameters as functions of density at zero temperature for SkI3 force. There are two types of solutions of the self-consistent equations in symmetric nuclear matter, corresponding to FM and AFM ordering of neutron and proton spins. Due to proximity of FL amplitudes U_1 and U_3, the respective critical densities are very close to each other ($\varrho_c \approx 0.910\,\mathrm{fm}^{-3}$ for FM ordering and $\varrho_c \approx 0.917\,\mathrm{fm}^{-3}$ for AFM ordering) and larger than the critical density of spin instability in neutron matter ($\varrho_c \approx 0.37\,\mathrm{fm}^{-3}$). When some admixture of protons is added to neutron matter, the last critical density is shifted to larger densities and a spin polarized state with the oppositely directed spins of neutrons and protons appears. Under increasing density of nuclear matter, the neutron spin polarization continuously increases till all neutron spins will be aligned in the same direction. Protons, at first, become more polarized with density and their spin polarization is opposite to the spin polarization of neutrons. But, after reaching the maximum, spin polarization of protons decreases and at some critical density spins of protons change direction, so that the spin ordered phase with the like-directed spins of neutrons and protons is formed. Then, beyond the critical density, the spin polarization of protons is continuing to increase until the totally polarized state with parallel ordering of neutron and proton spins will be formed. Thus, for SkI3 force nuclear matter undergoes at some critical density a phase transition from the state with antiparallel ordering of neutron and proton spins to the state with parallel ordering of spins. With increasing isospin asymmetry, this critical density increases as well. Note that there are no long tails in the density profiles of the neutron spin polarization parameter at large asymmetries.

In Fig. 3b the total energies per nucleon of spin ordered and nonpolarized states are compared at zero temperature for SkI3 force. In symmetric nuclear matter, FM spin ordering is thermodynamically more preferable than AFM one. The energy gain of the transition from the nonpolarized state to a spin polarized state increases with isospin asymmetry at the given density, analogously to SkI5 force. Hence, a spin polarized state in neutron matter occurs earlier in density than in symmetric nuclear matter, as was clarified above. Note that for SkI3 force under increasing density, initially, the state with antiparallel ordering of neutron and proton spins appears without existence of long tails in the density profiles of the neutron spin polarization parameter at strong isospin asymmetry. This is in contrast with SLy4 force, for which the antiparallel ordering at $\alpha \lesssim 1$ is characterized by the appearance of such long tails. Hence, the presence of long tails in the density profiles of the neutron spin polarization parameter doesn't relate to the antiparallel ordering of neutron and proton spins, but is associated with the decreasing dependence of the energy gain of the phase transition to a spin polarized state as a function of isospin asymmetry. In this case, the critical density of spin instability in symmetric nuclear matter is less than the critical density in neutron matter. If these critical densities are substantially different, that is the case for SLy4 force, then the long tails in the density profiles of the neutron spin polarization parameter appear.

In summary, we have considered spin ordered phase transitions in nuclear matter with SLy4, SkI3 and SkI5 Skyrme effective forces. It has been shown that asymmetric nuclear matter with SLy4 effective interaction undergoes a phase transition to a state with the oppositely directed spins of neutrons and protons. This phase transition is characterized by the appearance of long tails in the density profiles of the neutron spin polarization parameter near the transition point at strong isospin asymmetry. This means, that even small admixture of protons to neutron matter leads to the considerable shift of the critical density of spin instability to lower densities. The presence of such long tails is associated with the decreasing dependence of the energy gain of transition to a spin polarized state as a function of isospin asymmetry.

In nuclear matter with SkI5 effective interaction a spin polarized state with the like-directed spins of neutrons and protons is formed. For SkI3 force, nuclear matter under increasing density, at first, undergoes a phase transition to a spin polarized state with the oppositely directed spins of neutrons and protons. Under further increasing density, spins of protons change their direction at some critical point and a phase transition from the state with antiparallel ordering to the state with parallel ordering of neutron and proton spins occurs. Since the energy gain of the transition from the nonpolarized state to a polarized one is increased with isospin asymmetry, there are no long tails in the density distribution of the neutron spin polarization parameter for SkI3 and SkI5 forces. Note that the obtained results may be of importance for the adequate description of thermal and magnetic properties of neutron stars.

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